

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Suppose the population in California move outside of the state at a rate of 4% per year and that people from outside the state move in at a rate of 3% per year. Assuming the population inside and outside the state remain constant, what is the distribution of the population in California k years later? What is the distribution of the population as k becomes infinite?

Let the population at time $t = 0$ inside and outside be z_0 and y_0 respectively.

- a) Write a difference equation, Matrix equation, for the population inside and outside California after one year.
- b. Find a matrix P that diagonalizes the matrix in part a.
- e) Find an expression for (y_k, z_k) and discuss $\lim_{k \rightarrow \infty} (y_k, z_k)$

2. Solve the given system of linear equations initial value problem.

$$\begin{cases} \frac{dy_1}{dt} = -0.2y_1 \\ \frac{dy_2}{dt} = 0.2y_1 - 0.1y_2, & y_1(0) = 1, y_2(0) = 1, y_3(0) = 1, \\ \frac{dy_3}{dt} = 0.1y_3 \end{cases}$$